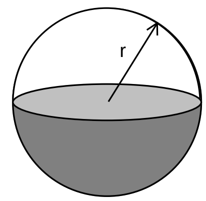
ÍNDICE

FORMULARIO DE MATEMÁTICAS

Geometría

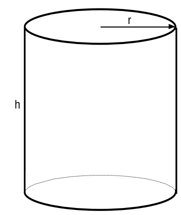
Volumen $= \frac{4}{3}\pi r^3$

Área de la Superficie $= 4\pi\, r^2$



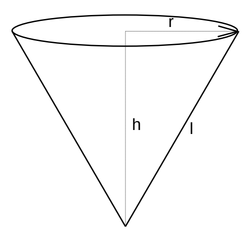
Volumen $= \pi r^2 h$

Área de la superficie lateral $= 2\pi r h$



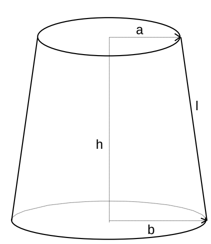
Volumen $= \frac{1}{3}\pi r^2 h$

Área de la superficie lateral $= \pi r\sqrt{r^2 + h^2} = \pi r\, l$



Volumen $= \frac{1}{3}\pi\, h\left(a^2 + a\,b + b^2\right)$

Área de la superficie lateral $\begin{aligned} &= \pi\left(a+b\right)\sqrt{h^2 + \left(b-a\right)^2} \\ &= \pi\left(a+b\right)l \end{aligned}$



Trigonometría

$\operatorname{sen}^2 A + \cos^2 A = 1$

$\sec^2 A - \tan^2 A = 1$

$\csc^2 A - \cot^2 A = 1$

$\tan A = \dfrac{\operatorname{sen} A}{\cos A}$

$\cot A = \dfrac{\cos A}{\operatorname{sen} A}$

$\operatorname{sen} A \csc A = 1$

$\cos A \sec A = 1$

$\tan A \cot A = 1$

$\operatorname{sen}(-A) = -\operatorname{sen} A$

$\cos(-A) = \cos A$

$\tan(-A) = -\tan A$

$\operatorname{sen}^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

$\operatorname{sen} 2A = 2\operatorname{sen} A \cos A$

$\cos 2A = \cos^2 A - \operatorname{sen}^2 A$

$\operatorname{sen}(A \pm B) = \operatorname{sen} A \cos B \pm \cos A \operatorname{sen} B$

$\cos(A \pm B) = \cos A \cos B \mp \operatorname{sen} A \operatorname{sen} B$

$tan\,(A \pm B) = \dfrac{tanA \pm tanB}{1 \mp tanAtanB}$

$\operatorname{sen}\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

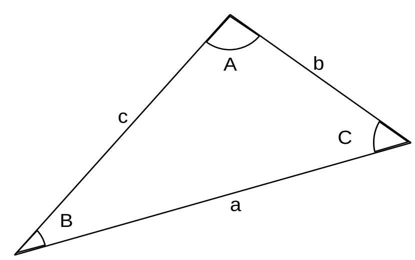
$\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

$\operatorname{sen} A \operatorname{sen} B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\operatorname{sen} A \cos B = \frac{1}{2}\left[\operatorname{sen}(A-B) + \operatorname{sen}(A+B)\right]$

$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

Las leyes siguientes son validas para cualquier triángulo plano ABC de lados a, b, c y de ángulos A, B, C.



Ley de los senos

$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{c}{\operatorname{sen} C}$$

Ley de los cosenos

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Los otros lados y ángulos están relacionados en forma similar

Ley de las tangentes

$$\frac{a+b}{a-b} = \frac{tan\,\frac{1}{2}(A+B)}{tan\,\frac{1}{2}(A-B)}$$

Los otros lados y ángulos están relacionados en forma similar

Números Complejos

Siendo p un número real cualquiera, el teorema de De Moivre establece que

$$\left[r(\cos\theta + i\operatorname{sen}\theta)\right]^p = r^p\left(\cos p\theta + i\operatorname{sen} p\theta\right)$$

Sea n cualquier entero positivo y $p = 1/n$, entonces

$$\left[r(\cos\theta + i\operatorname{sen}\theta)\right]^{1/n} = r^{1/n}\left[\cos\frac{\theta+2k\pi}{n} + i\operatorname{sen}\frac{\theta+2k\pi}{n}\right]$$

donde k es un entero positivo. De aquí se pueden obtener las n raíces n-ésimas distintas de un número complejo haciendo $k = 0,1,2,\cdots,n-1$

Geometría Analítica del Espacio

Considerando $P_1 = (x_1, y_1, z_1)$ y $P_2 = (x_2, y_2, z_2)$

Vector que une P_1 y P_2 :

$$\overline{P_1P_2} = \langle (x_2 - x_1), (y_2 - y_1), (z_2 - z_1) \rangle = (l,\, m,\, n)$$

Distancia entre dos puntos:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{l^2 + m^2 + n^2}$$

Recta que pasa por dos puntos:

- Forma Paramétrica:

$$x = x_1 + l\,t \qquad y = y_1 + m\,t \qquad z = z_1 + n\,t$$

-Forma Simétrica:

$$t = \frac{x - x_1}{l} \qquad t = \frac{y - y_1}{m} \qquad t = \frac{z - z_1}{n}$$

Cosenos Directores:

$$\cos\alpha = \frac{x_2 - x_1}{d} = \frac{l}{d} \qquad \cos\beta = \frac{y_2 - y_1}{d} = \frac{m}{d} \qquad \cos\gamma = \frac{z_2 - z_1}{d} = \frac{n}{d}$$

donde $\alpha,\ \beta,\ \gamma$ denotan los ángulos que forman la línea que une los puntos P_1 y P_2 con la parte positiva de los ejes x, y, z respectivamente.

Ecuación del Plano:

- Que pasa por un punto $P_1(x_1, y_1, z_1)$ y tiene vector normal $\vec{a} = \langle a_1, a_2, a_3 \rangle$:

$$a_1(x - x_1) + a_2(y - y_1) + a_3(z - z_1) = 0$$

-Forma General:

$$Ax + By + Cz + D = 0$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \text{o} \quad l^2 + m^2 + n^2 = 1$$

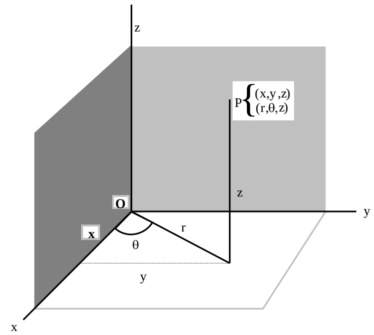
Distancia del punto $P_0(x_0, y_0, z_0)$ al plano Ax+By+Cz+D=0

$$d = \frac{Ax_0 + By_0 + Cz_0 + D}{\pm\sqrt{A^2 + B^2 + C^2}}$$

en la cual el signo debe escogerse de tal manera que la distancia no resulte negativa.

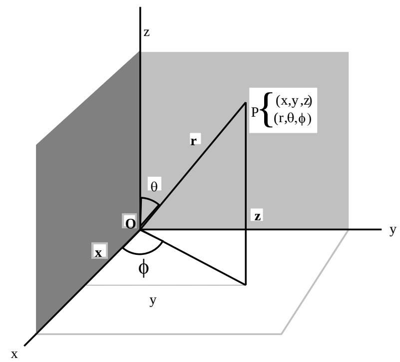
Coordenadas cilíndricas:

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ z = z \end{cases} \quad \text{o} \quad \begin{cases} r = \sqrt{x^2+y^2} \\ \theta = tan^{-1}\left(\frac{y}{x}\right) \\ z = z \end{cases}$$



Coordenadas esféricas:

$$\begin{cases} x = r\ \text{sen}\,\theta\ \cos\phi \\ y = r\ \text{sen}\,\theta\ \text{sen}\,\phi \\ z = r\ \cos\theta \end{cases} \quad \text{o} \quad \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \phi = tan^{-1}\left(\frac{y}{x}\right) \\ \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{cases}$$



Ángulo entre dos rectas en el plano $\tan\alpha = \dfrac{m_2 - m_1}{1+m_1m_2}$

Reglas Generales de Derivación

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(cx) = c$$

$$\frac{d}{dx}\left(cx^n\right) = ncx^{n-1}$$

$$\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \cdots$$

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(du/dx\right) - u\left(dv/dx\right)}{v^2}$$

$$\frac{d}{dx}\left(u^n\right) = nu^{n-1}\frac{du}{dx}$$

$$\frac{dF}{dx} = \frac{dF}{du}\frac{du}{dx}$$ (Regla de la cadena)

$$\frac{du}{dx} = \frac{1}{dx/du}$$

$$\frac{dF}{dx} = \frac{dF/du}{dx/du}$$

Derivadas de las Funciones Exponenciales y Logarítmicas

$$\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx} \qquad a>0, \quad a\neq 1$$

$$\frac{d}{dx}\ln u = \frac{d}{dx}\log_e u = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx}u^v = \frac{d}{dx}e^{v\ln u} = e^{v\ln u}\frac{d}{dx}\; v\ln u \; = vu^{v-1}\frac{du}{dx} + u^v \ln u \frac{dv}{dx}$$

Derivadas de las Funciones Trigonométricas y de las Trigonométricas Inversas

$$\frac{d}{dx}\operatorname{sen} u = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\operatorname{sen} u \frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sen}^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad -\tfrac{\pi}{2} < \operatorname{sen}^{-1} u < \tfrac{\pi}{2}$$

$$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad 0 < \cos^{-1} u < \pi$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx} \qquad -\tfrac{\pi}{2} < \tan^{-1} u < \tfrac{\pi}{2}$$

$$\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx} \qquad 0 < \cot^{-1} u < \pi$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} +si & 0<\sec^{-1} u < \tfrac{\pi}{2} \\ -si & \tfrac{\pi}{2} < \sec^{-1} u < \pi \end{bmatrix}$$

$$\frac{d}{dx}\csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\mp 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} -si & 0<\csc^{-1} u < \tfrac{\pi}{2} \\ +si & -\tfrac{\pi}{2} < \csc^{-1} u < 0 \end{bmatrix}$$

Derivadas de las Funciones Hiperbólicas y de las Hiperbólicas Recíprocas

$$\frac{d}{dx}\operatorname{senh} u = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx}\cosh u = \operatorname{senh} u \frac{du}{dx}$$

$$\frac{d}{dx}\tanh u = \operatorname{sech}^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\coth u = -\operatorname{csch}^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \, \tanh u \frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \, \coth u \frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sen h}^{-1}u=\frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$$

$$\frac{d}{dx}\cosh^{-1}u=\frac{\pm 1}{\sqrt{u^2-1}}\frac{du}{dx}\qquad\begin{bmatrix}+ & si & \cosh^{-1}u>0,u>1\\ - & si & \cosh^{-1}u<0,u<1\end{bmatrix}$$

$$\frac{d}{dx}\tanh^{-1}u=\frac{1}{1-u^2}\frac{du}{dx}\qquad -1<u<1$$

$$\frac{d}{dx}\coth^{-1}u=\frac{1}{1-u^2}\frac{du}{dx}\qquad\begin{bmatrix}u>1 & o & u<-1\end{bmatrix}$$

$$\frac{d}{dx}\operatorname{sec h}^{-1}u=\frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx}\qquad\begin{bmatrix}- & si & \operatorname{sec h}^{-1}u>0, & 0<u<1\\ + & si & \operatorname{sec h}^{-1}u<0, & 0<u<1\end{bmatrix}$$

$$\frac{d}{dx}\operatorname{csc h}^{-1}u=\frac{-1}{|u|\sqrt{1+u^2}}\frac{du}{dx}=\frac{\mp 1}{u\sqrt{1+u^2}}\frac{du}{dx}\qquad\begin{bmatrix}- & si & u>0, & + & si & u<0\end{bmatrix}$$

Tablas de Integrales

$$\int u\,dv=uv-\int v\,du$$

$$\int u^n du=\frac{1}{n+1}u^{n+1}+C\qquad n\neq -1$$

$$\int\frac{du}{u}=\ln|u|+C$$

$$\int e^u\,du=e^u+C$$

$$\int a^u du=\frac{a^u}{\ln a}+C$$

$$\int \operatorname{sen}u\,du=-\cos u+C$$

$$\int \cos u\,du=\operatorname{sen}u+C$$

$$\int \sec^2 u\,du=\tan u+C$$

$$\int \csc^2 u\,du=-\cot u+C$$

$$\int \sec u\tan u\,du=\sec u+C$$

$$\int \csc u\cot u\,du=-\csc u+C$$

$$\int \tan u\,du=\ln|\sec u|+C$$

$$\int \cot u\,du=\ln|\operatorname{sen}u|+C$$

$$\int \sec u\,du=\ln|\sec u+\tan u|+C$$

$$\int \csc u\,du=\ln|\csc u-\cot u|+C$$

$$\int\frac{du}{\sqrt{a^2-u^2}}=\operatorname{sen}^{-1}\frac{u}{a}+C$$

$$\int\frac{du}{a^2+u^2}=\frac{1}{a}\tan^{-1}\frac{u}{a}+C$$

$$\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}\sec^{-1}\frac{u}{a}+C$$

$$\int\frac{du}{a^2-u^2}=\frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right|+C$$

$$\int\frac{du}{u^2-a^2}=\frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|+C$$

$$\int\sqrt{a^2+u^2}\,du=\frac{u}{2}\sqrt{a^2+u^2}+\frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right|+C$$

$$\int u^2\sqrt{a^2+u^2}\,du=\frac{u}{8}\left(a^2+2u^2\right)\sqrt{a^2+u^2}-\frac{a^2}{8}\ln\left|u+\sqrt{a^2+u^2}\right|+C$$

$$\int\frac{du}{u\sqrt{a^2+u^2}}=-\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right|+C$$

$$\int\frac{du}{u^2\sqrt{a^2+u^2}}=-\frac{\sqrt{a^2+u^2}}{a^2u}+C$$

$$\int \frac{\sqrt{a^2+u^2}}{u}du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$$

$$\int \frac{\sqrt{a^2+u^2}}{u^2}du = -\frac{\sqrt{a^2+u^2}}{u} + \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{u^2du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{\left(a^2+u^2\right)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

$$\int \sqrt{a^2-u^2}\,du =$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}\left(2u^2-a^2\right)\sqrt{a^2-u^2} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u}du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u^2}du = -\frac{1}{u}\sqrt{a^2-u^2} - \operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{u^2du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2u}\sqrt{a^2-u^2} + C$$

$$\int \left(a^2-u^2\right)^{3/2}du = -\frac{u}{8}\left(2u^2-5a^2\right)\sqrt{a^2-u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{\left(a^2-u^2\right)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}\left(2u^2-a^2\right)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u}du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{u} + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u^2}du = -\frac{\sqrt{u^2-a^2}}{u} + \ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{u^2du}{\sqrt{u^2-a^2}} = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$$

$$\int \frac{du}{\left(u^2-a^2\right)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$$

$$\int \frac{udu}{a+bu} = \frac{1}{b^2}\left(a+bu-a\ln\left|a+bu\right|\right) + C$$

$$\int \frac{u^2du}{a+bu} = \frac{1}{2b^3}\left[\left(a+bu\right)^2 - 4a\left(a+bu\right) + 2a^2\ln\left|a+bu\right|\right] + C$$

$$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int \frac{u^2du}{\sqrt{a+bu}} = \frac{2}{15b^3}\left(8a^2+3b^2u^2-4abu\right)\sqrt{a+bu}$$

$$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C,\ \text{si}\ \ a>0$$

$$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C,\ \text{si}\ \ a<0$$

$$\int \frac{\sqrt{a+bu}}{u}du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$$

$$\int \frac{\sqrt{a+bu}}{u^2}du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$$

$$\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b}\ln|a+bu| + C$$

$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$$

$$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$$

$$\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}$$

$$\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$$

$$\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}du}{\sqrt{a+bu}}$$

$$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$$

$$\int \operatorname{sen}^2 u\,du = \tfrac{1}{2}u - \tfrac{1}{4}\operatorname{sen}2u + C$$

$$\int \cos^2 u\,du = \tfrac{1}{2}u + \tfrac{1}{4}\operatorname{sen}2u + C$$

$$\int \tan^2 u\,du = \tan u - u + C$$

$$\int \cot^2 u\,du = -\cot u - u + C$$

$$\int \operatorname{sen}^3 u\,du = -\tfrac{1}{3}\left(2+\operatorname{sen}^2 u\right)\cos u + C$$

$$\int \cos^3 u\,du = \tfrac{1}{3}\left(2+\cos^2 u\right)\operatorname{sen} u + C$$

$$\int \tan^3 u\,du = \tfrac{1}{2}\tan^2 u + \ln|\cos u| + C$$

$$\int \cot^3 u\,du = -\tfrac{1}{2}\cot^2 u - \ln|\operatorname{sen} u| + C$$

$$\int \sec^3 u\,du = \tfrac{1}{2}\sec u\,tanu + \tfrac{1}{2}\ln|\sec u + tanu| + C$$

$$\int \operatorname{sen} au\,\cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$\int u\operatorname{sen} u\,du = \operatorname{sen} u - u\cos u + C$$

$$\int u\cos u\,du = \cos u + u\operatorname{sen} u + C$$

$$\int u^n\operatorname{sen} u\,du = u^n\cos u + n\int u^{n-1}\cos u\,du$$

$$\int \csc^3 u\,du = -\tfrac{1}{2}\csc u\cot u + \tfrac{1}{2}\ln|\csc u - \cot u| + C$$

$$\int \operatorname{sen}^n u\,du = -\tfrac{1}{n}\operatorname{sen}^{n-1}u\cos u + \frac{n-1}{n}\int \operatorname{sen}^{n-2}u\,du$$

$$\int \cos^n u\,du = \tfrac{1}{n}\cos^{n-1}u\operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2}u\,du$$

$$\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1}u - \int \tan^{n-2}u\,du$$

$$\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1}u - \int \cot^{n-2}u\,du$$

$$\int \sec^n u\,du = \frac{1}{n-1}tanu\sec^{n-2}u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du$$

$$\int \csc^n u\,du = \frac{1}{n-1}\cot u\csc^{n-2}u + \frac{n-2}{n-1}\int \csc^{n-2}u\,du$$

$$\int \operatorname{sen} au\,\operatorname{sen} bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$\int \cos au\,\cos bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$\int u^n\cos u\,du = u^n\operatorname{sen} u - n\int u^{n-1}\operatorname{sen} u\,du$$

$$\int \operatorname{sen}^n u\cos^m u\,du$$
$$= -\frac{\operatorname{sen}^{n-1}u\cos^{m+1}u}{n+m} + \frac{n-1}{n+m}\int \operatorname{sen}^{n-2}u\cos^m u\,du$$
$$= -\frac{\operatorname{sen}^{n+1}u\cos^{m-1}u}{n+m} + \frac{m-1}{n+m}\int \operatorname{sen}^{n}u\cos^{m-2} u\,du$$

$$\int u\cos^{-1}u\,du = \frac{2u^2-1}{4}\cos^{-1}u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int u\tan^{-1}u\,du = \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C$$

$$\int \operatorname{sen}^{-1} u\,du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$$

$$\int \cos^{-1} u\,du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

$$\int \tan^{-1} u\,du = u \tan^{-1} u - \tfrac{1}{2}\ln\left(1+u^2\right) + C$$

$$\int u \operatorname{sen}^{-1} u\; du = \frac{2u^2-1}{4}\operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int u e^{au}\, du = \frac{1}{a^2}(au-1)e^{au} + C$$

$$\int u^n e^{au}\, du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}du$$

$$\int e^{au}\operatorname{sen} bu\; du = \frac{e^{au}}{a^2+b^2}(a\operatorname{sen} bu - b\cos bu) + C$$

$$\int e^{au}\cos bu\; du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\operatorname{sen} bu) + C$$

$$\int u^n \operatorname{sen}^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\operatorname{sen}^{-1} u - \int \frac{u^{n+1}du}{\sqrt{1-u^2}}\right],\quad n \neq -1$$

$$\int u^n \cos^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1}du}{\sqrt{1-u^2}}\right],\quad n \neq -1$$

$$\int u^n \tan^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1}du}{\sqrt{1+u^2}}\right], n \neq -1$$

$$\int \ln u\,du = u\ln u - u + C$$

$$\int u^n \ln u\; du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$$

$$\int \frac{1}{u\ln u}du = \ln\left|\ln u\right| + C$$

$$\int \operatorname{senh} u\,du = \cosh u + C$$

$$\int \cosh u\,du = \operatorname{senh} u + C$$

$$\int \tanh u\,du = \ln\cosh u + C$$

$$\int \coth u\,du = \ln\left|\operatorname{senh} u\right| + C$$

$$\int \operatorname{sech} u\,du = tan^{-1}\left|\operatorname{senh} u\right| + C$$

$$\int \operatorname{sech} u\,du = \ln\left|\tan \tfrac{1}{2}u\right| + C$$

$$\int \operatorname{sech}^2 u\,du = \tanh u + C$$

$$\int \operatorname{csch}^2 u\,du = -\coth u + C$$

$$\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$$

$$\int \sqrt{2au-u^2}\; du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int u\sqrt{2au-u^2}\; du = \frac{2u-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{\sqrt{2au-u^2}}{u^2}du = \sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{\sqrt{2au-u^2}}{u^2}du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{u^2 du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$

Vectores

$$\mathbf{A} \bullet \mathbf{B} = \|A\|\|B\| \cos\theta \qquad 0 \le \theta \le \pi$$

donde θ es el ángulo formado por **A** y **B**

$$\mathbf{A} \bullet \mathbf{B} = A_1 B_1 + A_2 B_2 + A_3 B_3$$

donde $\mathbf{A} = A_1 \hat{\mathbf{i}} + A_2 \hat{\mathbf{j}} + A_3 \hat{\mathbf{k}}$, $\mathbf{B} = B_1 \hat{\mathbf{i}} + B_2 \hat{\mathbf{j}} + B_3 \hat{\mathbf{k}}$

Son resultados fundamentales:

Producto cruz: $\mathbf{AxB} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$

$$= (A_2 B_3 - A_3 B_2)\hat{\mathbf{i}} + (A_3 B_1 - A_1 B_3)\hat{\mathbf{j}} + (A_1 B_2 - A_2 B_1)\hat{\mathbf{k}}$$

Magnitud del Producto Cruz $\|\mathbf{AxB}\| = \|\mathbf{A}\|\|\mathbf{B}\| \operatorname{sen}\theta$

El operador *nabla* se define así:

$$\nabla = \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial y} + \hat{\mathbf{k}} \frac{\partial}{\partial z}$$

En las fórmulas que vienen a continuación vamos a suponer que $U=U(x,y,z)$, y $\mathbf{A}=\mathbf{A}(x,y,z)$ tienen derivadas parciales.

Gradiente de U = grad U $= \nabla U = \left(\hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial y} + \hat{\mathbf{k}} \frac{\partial}{\partial z} \right) U = \frac{\partial U}{\partial x} \hat{\mathbf{i}} + \frac{\partial U}{\partial y} \hat{\mathbf{j}} + \frac{\partial U}{\partial z} \hat{\mathbf{k}}$

Divergencia de **A** = div **A** $= \nabla \bullet \mathbf{A} = \left(\hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial y} + \hat{\mathbf{k}} \frac{\partial}{\partial z} \right) \bullet \left(A_1 \hat{\mathbf{i}} + A_2 \hat{\mathbf{j}} + A_3 \hat{\mathbf{k}} \right)$

$$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

Rotacional de **A** = rot **A** $= \nabla \mathbf{x} \mathbf{A} = \left(\hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial y} + \hat{\mathbf{k}} \frac{\partial}{\partial z} \right) \mathbf{x} \left(A_1 \hat{\mathbf{i}} + A_2 \hat{\mathbf{j}} + A_3 \hat{\mathbf{k}} \right)$

$$= \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= \left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z} \right) \hat{\mathbf{i}} + \left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x} \right) \hat{\mathbf{j}} + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} \right) \hat{\mathbf{k}}$$

Laplaciano de U = $\nabla^2 U = \nabla \bullet (\nabla U) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$

Integrales Múltiples

$$\int_{x=a}^{b}\int_{y=f_1(x)}^{f_2(x)} F(x,y)\,dy\,dx$$
$$=\int_{x=a}^{b}\left\{\int_{y=f_1(x)}^{f_2(x)} F(x,y)\,dy\right\}dx$$

donde $y = f_1(x)$ e $y = f_2(x)$ son las ecuaciones de las curvas HPG y PGQ respectivamente, mientras que a y b son las abscisas de los puntos P y Q. Esta integral también se puede escribir así:

$$\int_{y=c}^{d}\int_{x=g_1(y)}^{g_2(y)} F(x,y)\,dx\,dy = \int_{y=c}^{d}\left\{\int_{x=g_1(y)}^{g_2(y)} F(x,y)\,dx\right\}dy$$

donde $x = g_1(y)$, $x = g_2(y)$ son las ecuaciones de las curvas HPG y PGQ respectivamente, mientras que c y d son las ordenadas de H y G.

Estas son las llamadas integrales dobles o integrales de área. Los anteriores conceptos se pueden ampliar para considerar integrales triples o de volumen así como integrales múltiples en más de tres dimensiones.

$$s = s(t) = \int_a^t \|\vec{r}\,'(t)\|\,dt$$

Es la longitud de curva correspondiente al intervalo paramétrico a,t .

	En parámetro arbitrario:	En parámetro s:
Vector tangente unitario	$\vec{t}\,(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$	$\vec{t}\,(s) = \dot{\vec{r}}\,(s)$
Vector normal principal	$\vec{n}(t) = \vec{b}(t)\mathbf{x}\,\vec{t}\,(t)$	$\vec{n}(s) = \dfrac{\ddot{\vec{r}}\,(s)}{\|\ddot{\vec{r}}\,(s)\|}$
Vector binormal	$\vec{b}(t) = \dfrac{\vec{r}\,'\mathbf{x}\vec{r}\,''(t)}{\|\vec{r}\,'\mathbf{x}\,\vec{r}\,''(t)\|}$	$\vec{b}(s) = \dfrac{\dot{\vec{r}}\,(s)\mathbf{x}\ddot{\vec{r}}\,(s)}{\|\ddot{\vec{r}}\,(s)\|}$

Los vectores unitarios $\vec{t}\,,\vec{n},\vec{b}$ forman un triedo positivo $\left(\vec{b} = \vec{t}\,\mathrm{x}\vec{n}, \vec{n} = \vec{b}\,\mathrm{x}\vec{t}\,, \vec{t} = \vec{n}\,\mathrm{x}\vec{b}\right)$

Recta tangente en t_0

Ecuación vectorial:

$$\vec{r}(\lambda) = \vec{r}(t_0) + \lambda\vec{r}\,'(t_0)$$

Ecuación paramétrica

$$\frac{x-x_0}{x_0'} = \frac{y-y_0}{y_0'} = \frac{z-z_0}{x_0'}$$

Plano osculador $(\vec{t}\,,\vec{n})$ en t_0

Ecuación vectorial

$$\left(\vec{r}-\vec{r}(t_0)\right)\bullet\left(\vec{r}\,'(t_0)x\vec{r}\,''(t_0)\right) = 0$$

Ecuación paramétrica

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_0' & y_0' & z_0' \\ x_0'' & y_0'' & z_0'' \end{vmatrix} = 0$$

Curvatura y Torsión

$$\kappa(t)=\frac{\left\|\vec{r}\,'(t)\mathrm{x}\vec{r}\,''(t)\right\|}{\left\|\vec{r}\,'(t)\right\|^{3}} \qquad \tau(t)=\frac{\vec{r}\,'(t)\cdot\left(\vec{r}\,''(t)\mathrm{x}\vec{r}\,'''(t)\right)}{\left\|\vec{r}\,'(t)\mathrm{x}\vec{r}\,''(t)\right\|^{2}}$$

$$\kappa(s)=\left\|\ddot{\vec{r}}(s)\right\| \qquad \kappa=\frac{\left|f''(x)\right|}{[1+(f'(x))^{2}]^{3/2}}$$

Plano Normal

Ecuación vectorial:

$$\left(\vec{r}-\vec{r}\left(t_0\right)\right)\cdot\vec{r}\,'\left(t_0\right)=0$$

Ecuación paramétrica:

$$x_0'\left(x-x_0\right)+y_0'\left(y-y_0\right)+z_0'\left(z-z_0\right)=0$$

Plano Rectificante $\left(\vec{t},\vec{b}\right)$ en t_0

Ecuación vectorial:

$$\left(\vec{r}-\vec{r}\left(t_0\right)\right)\cdot\vec{n}\left(t_0\right)=0$$

Ecuación paramétrica:

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_0' & y_0' & z_0' \\ y_0'z_0''-y_0''z_0' & z_0'x_0''-z_0''x_0' & x_0'y_0''-x_0''y_0' \end{vmatrix}=0$$

Componentes Tangencial y Normal de la Aceleración

$$a_T=\vec{a}\cdot\vec{T}=\frac{\vec{v}\,.\,\vec{a}}{\left\|\vec{v}\right\|}$$

$$a_N=\vec{a}\,.\,\vec{N}=\frac{\left\|\vec{v}\ \ x\ \ \vec{a}\right\|}{\left\|\vec{v}\right\|}$$

Propiedades de la Divergencia

i) div ($\vec{F}$ + $\vec{G}$) = div ($\vec{F}$) +div ($\vec{G}$)

ii) div (ϕ $\vec{F}$) = ϕ div($\vec{F}$) + (grad ϕ) $\bullet$ $\vec{F}$

iii) div ($\vec{F}$ + $\vec{G}$) = G $\bullet$ $\left[\ \text{rot}\,(\vec{F})\ \right]$ - $\vec{F}$ $\bullet$ $\left[rot\ (\vec{G})\ \right]$

Transformada de Laplace

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$$

No	*f*(*t*)	*F*(*s*)
1	C (*constante*)	$\frac{C}{s}$
2	t^n	$\frac{n!}{s^{n+1}}$, $n = 0$ y $n \in \mathbb{N}$
3	t^n	$\frac{\Gamma(n+1)}{s^{n+1}}$, $n > -1$
4	e^{at}	$\frac{1}{s-a}$
5	$\operatorname{senh} at$	$\frac{a}{s^2-a^2}$
6	$\cosh at$	$\frac{s}{s^2-a^2}$
7	$\operatorname{sen} kt$	$\frac{k}{s^2+k^2}$
8	$\cos kt$	$\frac{s}{s^2+k^2}$
9	$e^{at} f(t)$	$F(s-a)$
10	$f(t-a)U(t-a)$	$e^{-as}F(s)$
11	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
12	$\frac{f(t)}{t}$	$\int_s^{\infty} F(p)dp$
13	$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots f^{(n-1)}(0)$
14	$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
15	$f * g = \int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$
16	$f(t)$. Función periódica de periodo T	$\frac{1}{1-e^{-sT}} \int_0^T f(t)e^{-st}dt$

Fórmulas misceláneas

Ecuaciones paramétricas de la cicloide para $t \in R$

$x = a(t - \operatorname{sen} t)$ $y = a(1 - \cos t)$

Trabajo $W = \int_a^b \vec{F} \bullet d\vec{r}$ $Comp_{\vec{b}}\vec{a} = \dfrac{\vec{a} \bullet \vec{b}}{\|\vec{b}\|}$

Longitud de arco de $y = f(x)$ **en** $[a,b] = \int_a^b \sqrt{1+(y')^2}\,dx$

$m = \iint_R \rho(x,y)dA$ $M_x = \iint_R y\rho(x,y)dA$ $M_y = \iint_R x\rho(x,y)dA$

Centro de gravedad de una región plana $\bar{x} = \dfrac{\int_a^b xf(x)dx}{\int_a^b f(x)dx}$, $\bar{y} = \dfrac{\frac{1}{2}\int_a^b [f(x)]^2 dx}{\int_a^b f(x)dx}$

Longitud de arco en forma paramétrica $L = \int_\alpha^\beta \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$

Momento de inercia de R respecto al origen $= I_o = \iint_R (x^2 + y^2)\rho(x,y)dA$

Área de la superficie generada al girar la gráfica f **alrededor de** x

$$S = \int_a^b 2\pi F(x)\sqrt{1+(f(x))^2}\,dx$$

Volumen del sólido de revolución generado al girar la gráfica de f **alrededor del eje** y

$$V = \int_a^b 2\ \pi t F(t)dt$$

Cálculo del volumen $V = \int_a^b A(x)dx$ $V = \int_a^b \pi(f(x))^2 dx$

Ecuación diferencial de primer orden $y' + P(x)y = Q(x)$

Solución $ye^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}dx + k$

Ecuación del resorte helicoidal $\vec{r}(t) = \left\langle \cos t, \operatorname{sen} t, \dfrac{t}{2\pi} \right\rangle$

Derivada direccional $D_{\vec{u}}\, f(x,y,z) = \nabla f(x,y,z) \bullet \vec{u}$ ($\vec{u}$ vector unitario)

Ecuación satisfecha por la carga de un circuito LRC $Lq'' + Rq' + \dfrac{1}{C}q = E(t)$

Fuerza ejercida por un fluído $F = \int_a^b \gamma y \cdot L(y)dy$

Fuerza que actúa sobre un líquido encerrado en un tubo $F = \delta\ A\,2x_0\,g - \delta\ A2xg$

Series de Fourier

Serie de Fourier para una función suave a tramos en [-L, L]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

Donde

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

Serie de Fourier para una función par en [-L, L]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

Donde $$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx \qquad a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

Serie de Fourier para una función impar en [-L, L]

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

Donde $$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

Serie de Fourier para una función definida en [0, L]

a) Serie de Cosenos $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

Donde $$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx \qquad a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

b) Serie de Senos $$f(x) = \sum_{n=1}^{\infty} b_n \cos\left(\frac{n\pi x}{L}\right)$$

Donde $$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

Serie Compleja de Fourier en [-L, L]

$$f(x) = \sum_{-\infty}^{\infty} C_n e^{\frac{in\pi x}{L}}$$

Donde $$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-\frac{in\pi x}{L}}\,dx$$